Matching Numbers in a Lottery

The number of ways of selecting 6 numbers from the numbers 1 through 49 (without replacement) is

$$\binom{49}{6} = \frac{49!}{6!(49-6)!} = 13,983,816$$

Now supose that 6 numbers have been selected as above by Player A. The number of ways that Player B may choose 6 numbers between 1 and 49 (without repetition) so that exactly one of the chosen numbers matches one of the six selected by Player A is

$$\binom{6}{1} \cdot \binom{43}{5} = 6 \cdot 962,598 = 5,775,588$$

The first factor is the number of ways Player B can select one number from the 6 selected by Player A. The second factor is the number of ways the 5 remaining numbers can be selected by Player B so that none matches the remaining 5 numbers chosen by Player A. Similarly, the number of ways exactly two of the six numbers chosen by Player B match those selected by Player A is

$$\binom{6}{2} \cdot \binom{43}{4} = 15 \cdot 123,410 = 1,851,150$$

The number of ways there are exactly three matches is

$$\binom{6}{3} \cdot \binom{43}{3} = 20 \cdot 12,341 = 246,820$$

The mumber of ways there are exactly four matches is

$$\binom{6}{4} \cdot \binom{43}{2} = 15 \cdot 903 = 13,545$$

The number of ways there are exactly five matches is

$$\binom{6}{5} \cdot \binom{43}{1} = 6 \cdot 43 = 258$$

And, of course, the number of ways all six match is 1. To compute probabilities of of the above events, just take quotients of the number of occurances of the events with $\binom{49}{6}$. For example, the probability of choosing exactly one correct number is

$$\frac{5,775,588}{13,983,816} \approx 0.413019$$